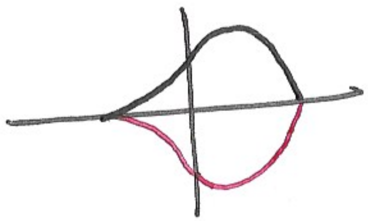
10.8 POLAR GRAPHS (SYMMETRY)

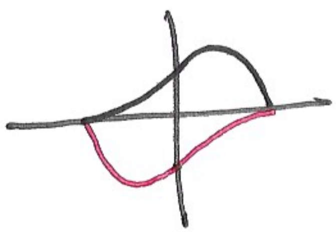
IF A GRAPH IS SYM OVER	WE ONLY NEED TO PLOT POINTS FOR θ IN
POLAR AXIS	Q_1, Q_2 $\theta \in [0, \pi]$
POLE	Q_1, Q_2 $\theta \in [0, \pi]$ OR Q_1, Q_4 $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
$\theta = \frac{\pi}{2}$	Q_1, Q_4 $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

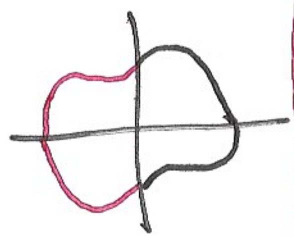
POLAR AXIS



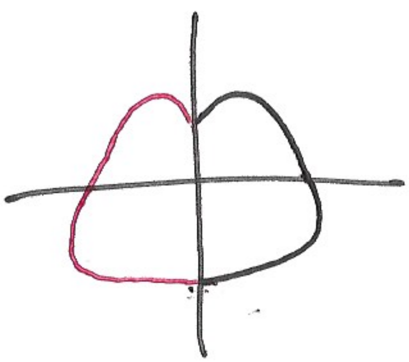
MINIMUM INTERVAL TO PLOT POINTS

POLE





$\theta = \frac{\pi}{2}$



BACK TO $r = \sin\theta$

SYM TESTS POLAR AXIS $\longrightarrow$ NO CONCLUSION

POLE $\longrightarrow$ NO CONCLUSION

$\theta = \frac{\pi}{2}$ $\longrightarrow$ SYM

SO WE NEED TO PLOT POINTS FOR $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

+ THEN REFLECT

THE CONCLUSION OF OUR POLAR SYM TESTS IS NEVER "NOT SYMMETRIC"

10.8 POLAR GRAPHS

NEED 16 POINTS IF PLOTTING POINTS ONLY

$r = \sin\theta$

FOR GRAPHING

$\sqrt{2} \approx 1.4$

$\sqrt{3} \approx 1.8$

θ	$r=\sin\theta$	(r,θ)
0	$\sin 0 = 0$	$(0,0)$ 5
$\frac{\pi}{6}$	$\sin\frac{\pi}{6} = \frac{1}{2}$	$\left(\frac{1}{2}, \frac{\pi}{6}\right) = \left(0.5, \frac{\pi}{6}\right)$ 6
$\frac{\pi}{4}$	$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right) \approx \left(0.7, \frac{\pi}{4}\right)$ 7
$\frac{\pi}{3}$	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right) \approx \left(0.9, \frac{\pi}{3}\right)$ 8
$\frac{\pi}{2}$	$\sin\frac{\pi}{2} = 1$	$\left(1, \frac{\pi}{2}\right)$ 9
$-\frac{\pi}{6}$ ~~$\frac{2\pi}{3}$~~	$\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$	$\left(-\frac{1}{2}, -\frac{\pi}{6}\right) = \left(-0.5, -\frac{\pi}{6}\right)$ 4
$-\frac{\pi}{4}$ ~~$\frac{3\pi}{4}$~~	$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$	$\left(-\frac{\sqrt{2}}{2}, -\frac{\pi}{4}\right) \approx \left(-0.7, -\frac{\pi}{4}\right)$ 3
$-\frac{\pi}{3}$ ~~$\frac{5\pi}{6}$~~	$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$	$\left(-\frac{\sqrt{3}}{2}, -\frac{\pi}{3}\right) \approx \left(-0.9, -\frac{\pi}{3}\right)$ 2
$-\frac{\pi}{2}$ ~~π~~	$\sin\left(-\frac{\pi}{2}\right) = -1$	$\left(-1, -\frac{\pi}{2}\right)$ 1
~~$\frac{7\pi}{6}$~~		
~~$\frac{5\pi}{4}$~~ ...		

$\sin(-x) = -\sin x$

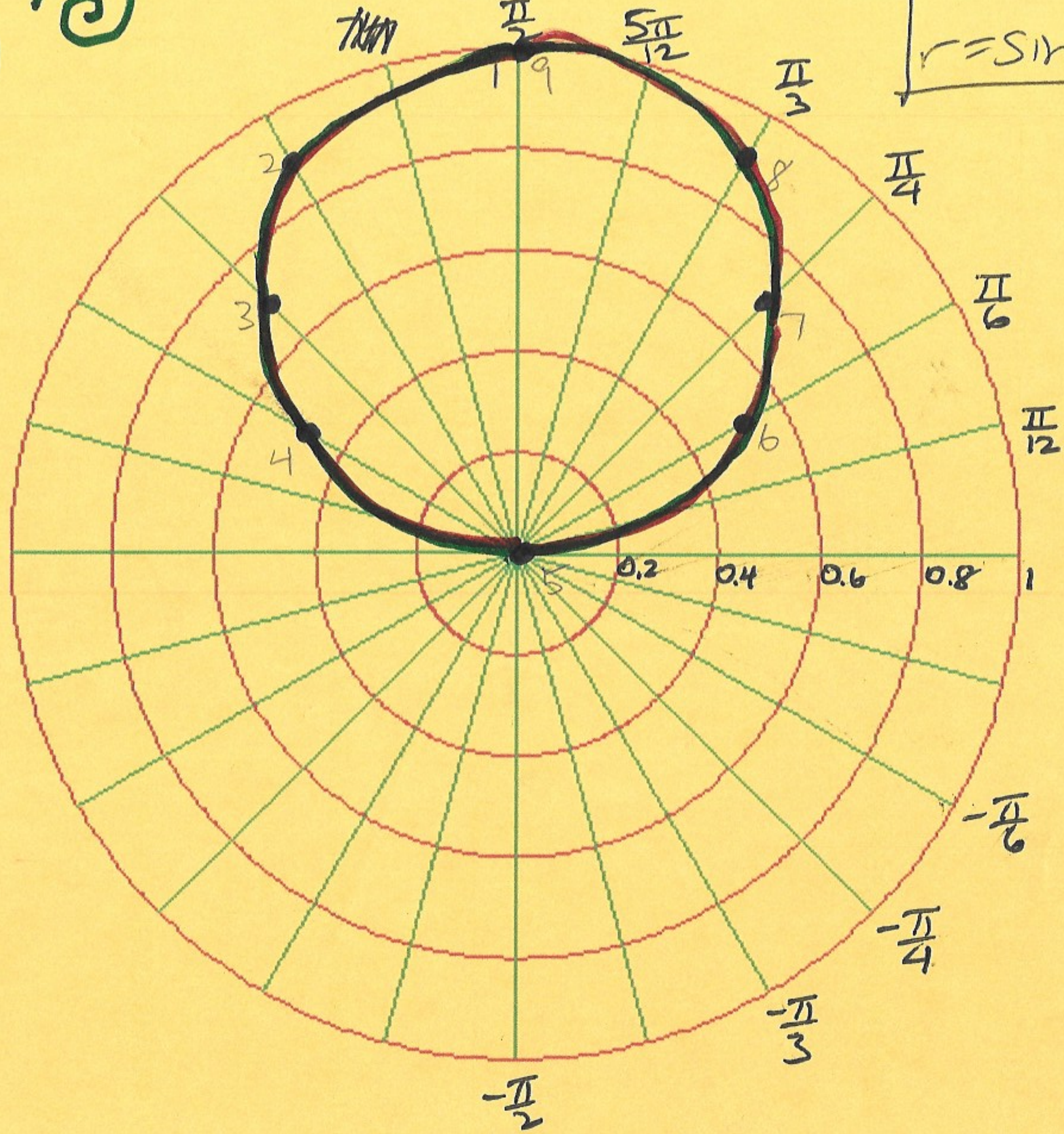
ALL IN Q_1

IN $Q_{\cancel{3}4}$

~~IN Q_3~~

NOTE: WHEN θ IN Q_4, r VALUES NEGATIVE, SO POINTS IN Q_2 (OPPOSITE DIRECTION)

$\boxed{r = \sin\theta}$



GRAPH $r = 1 + \cos 2\theta$ USING SYMMETRY

POLAR AXIS ✓ $(r, -\theta)$: $r = 1 + \cos 2(-\theta)$

$r = 1 + \cos(-2\theta)$

$r = 1 + \cos 2\theta \longleftarrow$ SYM

POLAR AXIS ✓$(r, -\theta)$ ~~$(-r, \pi-\theta)$~~

POLE ?~~$(-r, \theta)$~~ ✓$(r, \pi+\theta)$

$\theta = \frac{\pi}{2}$?$(-r, -\theta)$ $(r, \pi-\theta)$

$\cos(-x) = \cos x$

POLE ✓ $(r, \pi+\theta)$: $r = 1 + \cos 2(\pi+\theta)$

$r = 1 + \cos(2\pi + 2\theta)$

$r = 1 + \cancel{\cos 2\pi}^{1} \cos 2\theta - \cancel{\sin 2\pi}^{0} \sin 2\theta$

$r = 1 + \cos 2\theta \longleftarrow$ SYM

$\cos(A+B)$

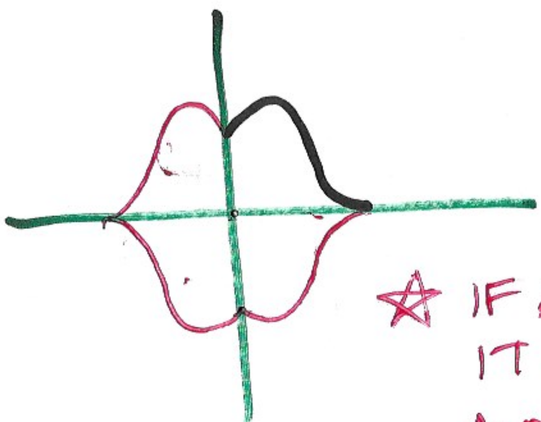
$= \cos A \cos B - \sin A \sin$

YOU COULD RUN SYM TEST TO DETERMINE IF GRAPH IS SYM OVER $\theta = \frac{\pi}{2}$

BUT, IF A GRAPH IS SYM OVER POLAR AXIS + POLE

IT IS AUTOMATICALLY SYM OVER $\theta = \frac{\pi}{2}$

SO, ~~WE~~ YOU DON'T NEED TO RUN THE SYM TESTS FOR $\theta = \frac{\pi}{2}$



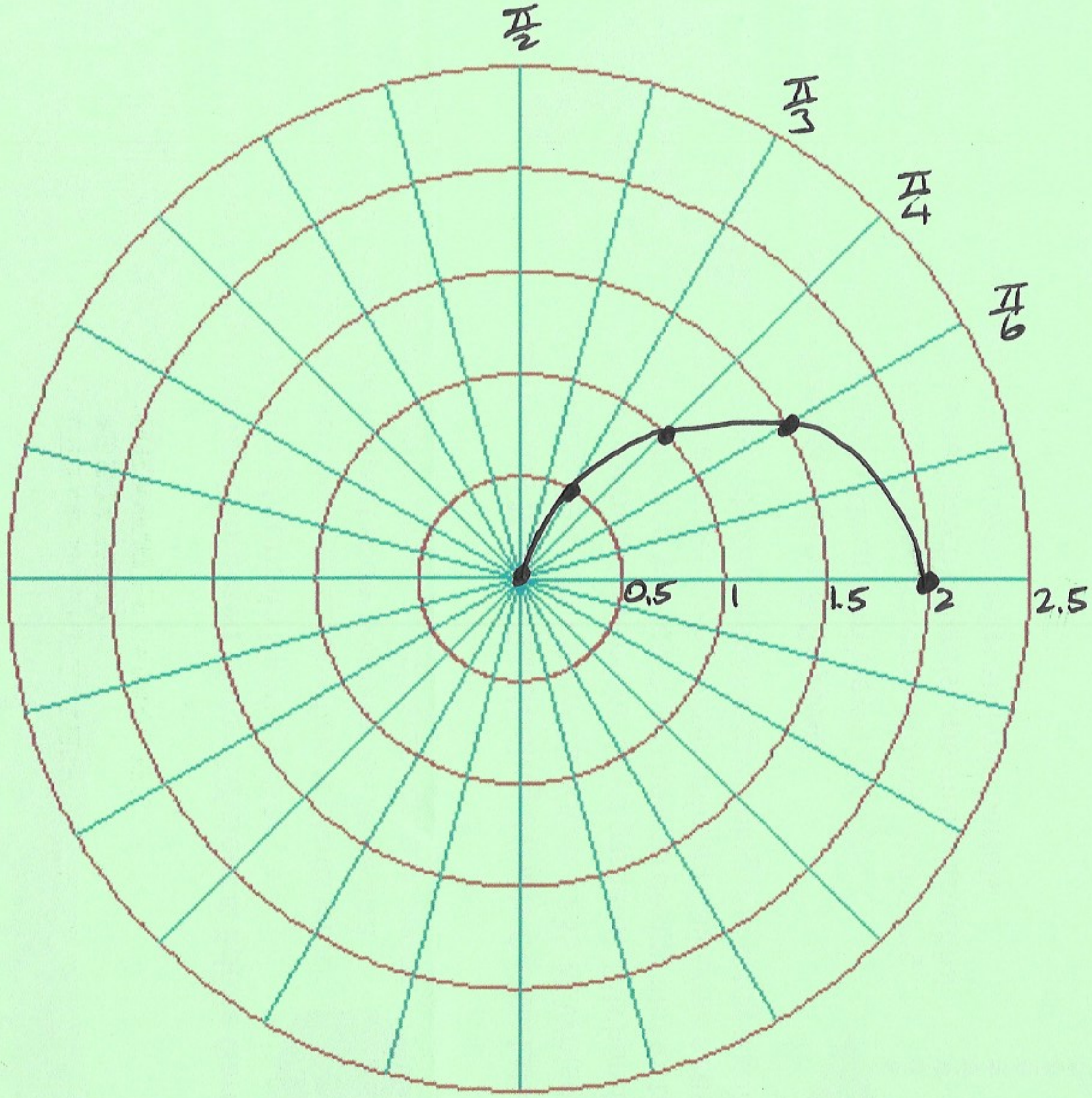
☆ IF A GRAPH IS SYM IN 2 WAYS,

IT IS AUTOMATICALLY SYMMETRIC IN ALL 3 W

AND THE MINIMUM INTERVAL IS $\theta \in [0, \frac{\pi}{2}]$

$r = 1 + \cos 2\theta$

θ	$r = 1 + \cos 2\theta$	(r, θ)
0	$1 + \cos 2(0) = 1 + \cos 0 = 1 + 1 = 2$	$(2, 0)$
$\frac{\pi}{6}$	$1 + \cos 2(\frac{\pi}{6}) = 1 + \cos \frac{\pi}{3} + 1 + \frac{1}{2} = 1\frac{1}{2}$	$(1.5, \frac{\pi}{6})$
$\frac{\pi}{4}$	$1 + \cos 2(\frac{\pi}{4}) = 1 + \cos \frac{\pi}{2} = 1 + 0 = 1$	$(1, \frac{\pi}{4})$
$\frac{\pi}{3}$	$1 + \cos 2(\frac{\pi}{3}) = 1 + \cos \frac{2\pi}{3} = 1 - \frac{1}{2} = \frac{1}{2}$	$(0.5, \frac{\pi}{3})$
$\frac{\pi}{2}$	$1 + \cos 2(\frac{\pi}{2}) = 1 + \cos \pi = 1 - 1 = 0$	$(0, \frac{\pi}{2})$



π/2
π/3
π/4
π/6
0.5
1
1.5
2
2.5